Statistical Natural Language Processing

Sequence learning

Çağrı Çöltekin

University of Tübingen
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Some (typical) machine learning applications

	x (input)	y (output)
Spam detection	document	spam or not
Sentiment analysis	product review	sentiment
Medical diagnosis	patient data	diagnosis
Credit scoring	financial history	loan decision

The cases (input–output) pairs are assumed to be *independent and identically distributed* (i.i.d.).

Structured prediction

In many applications, the i.i.d. assumption is wrong

	x (input)	y (output)
POS tagging	word sequence	POS sequence
Parsing	word sequence	parse tree
OCR	image (array of pixels)	sequences of letters
Gene prediction	genome	genes

Structured/sequence learning is prevalent in NLP.

Sequence learning in NLP: examples

tokenization

```
The U.N. is the largest intergovernmental
BIIOBIIIOBIOBIIOBIIIIIIOBIIIIIIIIIIIIIIIO
```

Sequence learning in NLP: examples

named-entity recognition

UN	Secretary-General	Antonio	Guterres
ORG	NONE	PER	PER
plans	to	visit	Ukraine
NONE	NONE	NONE	GEO

Sequence learning in NLP: examples

part of speech tagging

Time	flies	like	an	arrow	.
NOUN	VERB	ADP	DET	NOUN	PUNC

- In all of the examples,
  - word/character–label pairs are not independent of each other
  - we want to get the best label sequence — not the best label for each word independently

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In this lecture …

- Hidden Markov models (HMMs)
- A short note on graphical probabilistic models
- Alternatives to HMMs (briefly): HMEM / CRF

… and later

- Recurrent neural networks

Markov chains

A *Markov chain* is a process where probability of an event depends only on the previous event(s).

A Markov chain is defined by,

- A set of states $Q = \{q_1, \ldots, q_n\}$
- A special start state q_0
- A transition probability matrix

$$\mathbf{A} = \begin{bmatrix} a_{01} & a_{02} & \ldots & a_{0n} \\ a_{11} & a_{12} & \ldots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

where a_{ij} is the probability of transition from state i to state j

Markov chains

calculating probabilities

Given a sequence of events (or states), $q_1, q_2, \dots q_t$,

- In a *first-order* Markov chain probability of an event q_t is

$$P(q_t|q_1, \dots, q_{t-1}) = P(q_t|q_{t-1})$$

- Sometimes this equality is just an assumption (as in n-gram models)
- In higher order chains, the dependence of history is extended, e.g., second-order Markov chain:

$$P(q_t|q_t, \dots, q_{t-1}) = P(q_t|q_{t-2}, q_{t-1})$$

A relevant example of a Markov Chain is n-gram language models (coming soon).

Hidden/latent variables

- In many machine learning problems we want to account for unobserved/unobservable *latent* or *hidden* variables
- Some examples
  - 'personality' in many psychological data
  - 'topic' of a text
  - 'socio-economic class' of a speaker
- Latent variables make learning difficult: since we cannot observe them, how do we set the parameters?

Learning with hidden variables

An informal/quick introduction to the EM algorithm

- The EM algorithm (or its variants) is used in many machine learning models with latent/hidden variables

1. Randomly initialize the parameters
2. Iterate until convergence:

E-step compute likelihood of the data, given the parameters
M-step re-estimate the parameters using the predictions based on the E-step

Hidden Markov models (HMM)

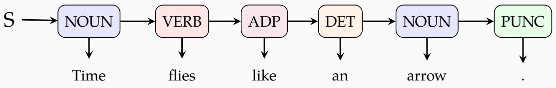
- HMMs are like Markov chains: probability of a state depends only a limited history of previous states

$$P(q_t|q_1, \dots, q_{t-1}) = P(q_t|q_{t-1})$$

- Unlike Markov chains, state sequence is hidden, they are not the observations
- At every state q_t, an HMM *emits* an output, o_t, whose probability depends only on the associated hidden state
- Given a state sequence $\mathbf{q} = q_1, \dots, q_T$, and the corresponding observation sequence $\mathbf{o} = o_1, \dots, o_T$,

$$P(\mathbf{o}, \mathbf{q}) = p(q_1) \left[\prod_2^T P(q_t|q_{t-1}) \right] \prod_1^T P(o_t|q_t)$$

Example: HMMs for POS tagging



- The tags are hidden
- Probability of a tag depends on the previous tag
- Probability of a word at a given state depends only on the current tag

HMMs: formal definition

An HMM is defined by

- A set of state $Q = \{q_1, \dots, q_n\}$
- The set of possible observations $V = \{v_1, \dots, v_m\}$
- A transition probability matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

a_{ij} is the probability of transition from state q_i to state q_j

- Initial probability distribution $\pi = \{P(q_1), \dots, P(q_n)\}$
- Probability distributions of

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

b_{ij} is the probability of emiting output o_i at state q_j

A simple example

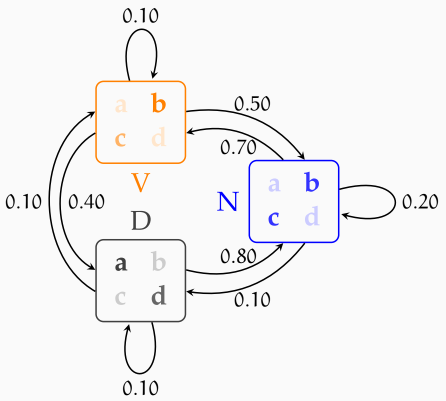
- Three states: `N`, `V`, `D`
- Four possible observations: `a`, `b`, `c` , `d`

$$\mathbf{A} = \begin{array}{c c c l} \mathtt{N} & \mathtt{V} & \mathtt{D} & \\ \left[\begin{array}{ccc} 0.2 & 0.7 & 0.1 \\ 0.5 & 0.1 & 0.4 \\ 0.8 & 0.1 & 0.1 \end{array} \right. & & \left. \right] & \begin{array}{l} \mathtt{N} \\ \mathtt{V} \\ \mathtt{D} \end{array} \end{array}$$

$$\mathbf{A} = \begin{array}{ccc|c} \mathtt{N} & \mathtt{V} & \mathtt{D} & \\ 0.2 & 0.7 & 0.1 & \mathtt{N} \\ 0.5 & 0.1 & 0.4 & \mathtt{V} \\ 0.8 & 0.1 & 0.1 & \mathtt{D} \end{array} \qquad \mathbf{B} = \begin{array}{ccc|c} \mathtt{N} & \mathtt{V} & \mathtt{D} & \\ 0.1 & 0.1 & 0.5 & \mathtt{a} \\ 0.4 & 0.5 & 0.1 & \mathtt{b} \\ 0.4 & 0.3 & 0.1 & \mathtt{c} \\ 0.1 & 0.1 & 0.3 & \mathtt{d} \end{array}$$

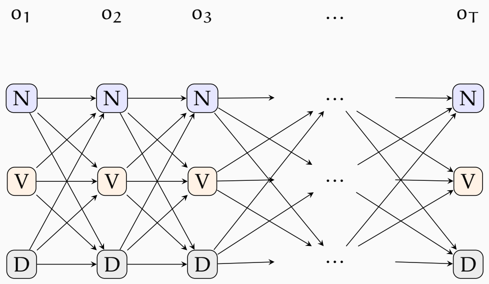
$$\boldsymbol{\pi} = (0.3, 0.1, 0.6)$$

HMM transition diagram



Unfolding the states

HMM lattice (or trellis)



HMMs: three problems

Evaluation

Calculating likelihood of a given sequence

$$P(\mathbf{o} \mid M)$$

Recognition/decoding

Calculating probability of state sequence, given an observation sequence

$$P(\mathbf{q} \mid \mathbf{o}; M)$$

Learning

Given observation sequences, a set of states, and (sometimes) corresponding state sequences, estimate the parameters ($\boldsymbol{\pi}$, $\mathbf{A}$, $\mathbf{B}$) of the HMM

Assigning probabilities to observation sequences

$$P(\mathbf{o} \mid M) = \sum_{\mathbf{q}} P(\mathbf{o}, \mathbf{q} \mid M)$$

- We need to sum over an exponential number of hidden state sequences
- The solution is using a dynamic programming algorithm
  - for each node of the trellis, store *forward probabilities*

$$\alpha_{t,i} = \sum_{j}^{N} \alpha_{t-1,j} P(q_i|q_j) P(o_i|q_i)$$

Assigning probabilities to observation sequences

the forward algorithm

- Start with calculating all forward probabilities for $t = 1$

$$\alpha_{1,i} = \pi_i P(o_1|q_i) \quad \text{for } 1 \leqslant i \leqslant N$$

store the α values

- For $t > 1$,

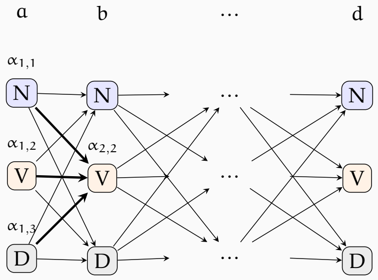
$$\alpha_{t,i} = \sum_{j=1}^{N} \alpha_{t-1,j} P(q_i|q_j) P(o_i|q_i) \quad \text{for } 1 \leqslant i \leqslant N, 2 \leqslant t \leqslant T$$

- Likelihood of the observation is the sum of the forward probabilities of the last step

$$P(\mathbf{o}|M) = \sum_{i=1}^{N} \alpha_{i,T}$$

Forward algorithm

HMM lattice (or trellis)



$$\alpha_{1,1} = \pi_N b_{aN}$$

$$\alpha_{2,2} = \alpha_{1,1} a_{NV} b_{bV} + \alpha_{1,2} a_{VV} b_{bV} + \alpha_{1,3} a_{DV} b_{bV}$$

Determining best sequence of latent variables

Decoding

- We often want to know the hidden state sequence given an observation sequence, $P(\mathbf{q} \mid \mathbf{o}; M)$
  - For example, given a sequence of tokens, find the most likely POS tag sequence
- The problem (also the solution, the *Viterbi algorithm*) is very similar to the forward algorithm
- Two major differences
  - we store maximum likelihood leading to each node on the lattice
  - we also store backlinks, the previous state that leads to the maximum likelihood

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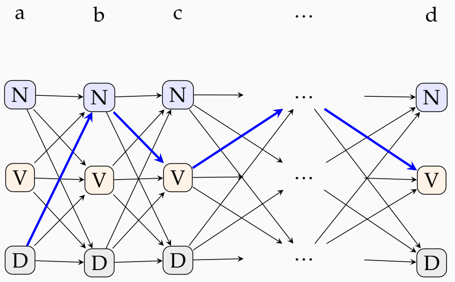
HMM decoding problem



Learning the parameters of an HMM

supervised case

- We want to estimate $\boldsymbol{\pi}$, $\mathbf{A}$, $\mathbf{B}$
- If we have both the observation sequence $\mathbf{o}$ and the corresponding state sequence, MLE estimate is

$$\pi_i = \frac{C(q_0 \to q_i)}{\sum_k C(q_0 \to q_k)}$$
$$a_{ij} = \frac{C(q_i \to q_j)}{\sum_k C(q_i \to q_k)}$$
$$b_{ij} = \frac{C(q_i \to o_j)}{\sum_k C(q_i \to o_k)}$$

Learning the parameters of an HMM

- Given a training set with observation sequence(s) $\mathbf{o}$ and state sequence $\mathbf{q}$, we want to find $\theta = (\boldsymbol{\pi}, \mathbf{A}, \mathbf{B})$

$$\arg\max_{\theta} P(\mathbf{o} \mid \mathbf{q}, \theta)$$

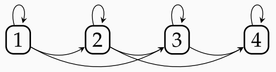
- Unlike i.i.d. case, we cannot factorize the likelihood over all observations
- Instead we use EM
  1. Initialize θ
  2. Repeat until convergence

  E-step given θ, estimate the hidden state sequence

  M-step given the estimated hidden states, use 'expected counts' to update θ
- An efficient implementation of EM algorithm is called *Baum-Welch algorithm*, or *forward-backward algorithm*

HMM variations

- The HMMs we discussed so far are called *ergodic* HMMs: all a_{ij} are non-zero
- For some applications, it is common to use HMMs with additional restrictions
- A well known variant (Bakis HMM) allows only forward transitions



- The emission probabilities can also be continuous, e.g., $p(q|o)$ can be a normal distribution

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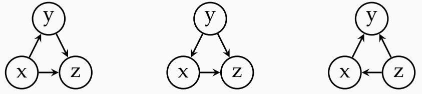
Directed graphical models: a brief divergence

Bayesian networks

- We saw earlier that joint distributions of multiple random variables can be factorized different ways

$$P(x, y, z) = P(x)P(y|x)P(z|x, y) = P(y)P(x|y)P(z|x, y) = P(z)P(x|z)P(y|x, z)$$

- *Graphical models* display this relations in graphs,
  - variables are denoted by nodes,
  - the dependence between the variables are indicated by edges
- Bayesian networks are directed acyclic graphs

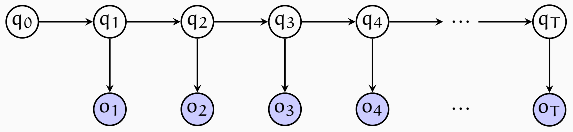


- A variable (node) depends only on its parents

Graphical models

- Graphical models define models involving multiple random variables
- It is generally more intuitive (compared to corresponding mathematical equations) to work with graphical models
- In a graphical model, by convention, the observed variables are shaded
- Graphs can also be undirected, which are called *Markov random fields*

HMM as a graphical model



MaxEnt HMMs (MEMM)

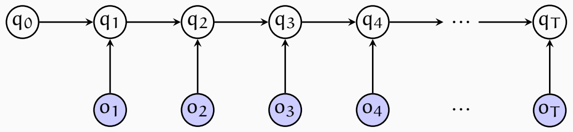
- In HMMs, we model $P(\mathbf{q}, \mathbf{o}) = P(\mathbf{q})P(\mathbf{o} \mid \mathbf{q})$
- In many applications, we are only interested in $P(\mathbf{q} \mid \mathbf{o})$, which we can calculate using the Bayes theorem
- But we can also model $P(\mathbf{q} \mid \mathbf{o})$ directly using a *maximum entropy model*

$$P(q_t \mid q_{t-1}, o_t) = \frac{1}{Z} e^{\sum w_i f_i(o_t, q_t)}$$

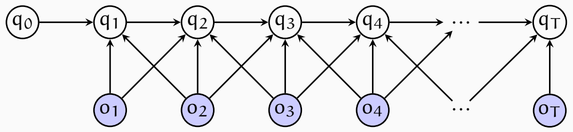
f_i are features – can be any useful feature

Z normalizes the probability distribution

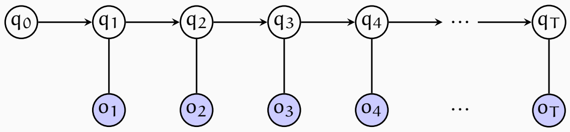
MEMMs as graphical models



We can also have other dependencies as features, for example



Conditional random fields



- A related model used in NLP is *conditional random field* (CRF)
- CRFs are *undirected models*
- CRFs also model $P(\mathbf{q} \mid \mathbf{o})$ directly

$$P(\mathbf{q} \mid \mathbf{o}) = \frac{1}{Z} \prod_t f(q_{t-1}, q_t) g(q_t, o_t)$$

Generative vs. discriminative models

- HMMs are *generative* models, they model the joint distribution
  - you can generate the output using HMMs
- MEMMs and CRFs are *discriminative* models they model the conditional probability directly
- It is easier to add arbitrary features on discriminative models
- In general: HMMs work well when the state sequence, $P(\mathbf{q})$, can be modeled well

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Summary

- In many problems, e.g., POS tagging, i.i.d. assumption is wrong
- We need models that are aware of the effects of the sequence (or structure in general) in the data
- HMMs are generative sequence models:
  - Markov assumption between the hidden states (POS tags)
  - Observations (words) are conditioned on the state (tag)
- There are other sequence learning methods
  - Briefly mentioned: MEMM, CRF
  - Coming soon: recurrent neural networks

Next

Mon	Unsuprevised learning
Wed	Pracical session / discussion of Assignment 2 and (maybe) sample exam questions from Monday
Fri	Neural networks